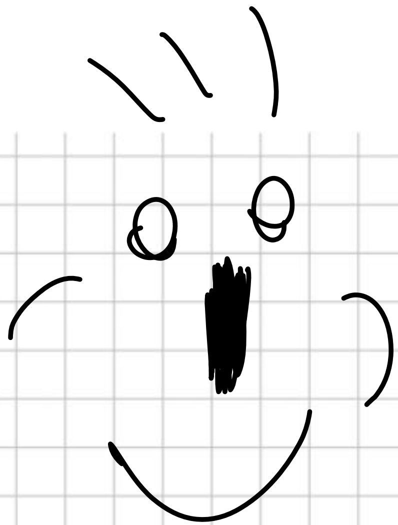
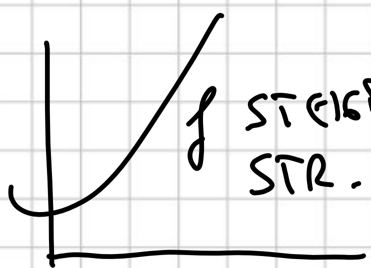


$y = f(x) = \frac{1}{12} \cdot x^3 - x^2 + 3x$

1) $D_f(x) = \mathbb{R}$ 2) $W_f(x) = \mathbb{R}$

3) STEIGUNGSVERHALTEN



f STEIGT
STR. MONOTON

$f'(x) > 0$

f STEIGT MONOTON

$$f(x) = \frac{1}{12} \cdot x^3 - x^2 + 3x$$

$$f'(x) = \frac{1}{4} \cdot x^2 - 2x + 3$$

$$\boxed{\left(a x^n\right)' = a \cdot n \cdot x^{n-1}}$$

$$f'(x) \overset{!}{=} 0 \Leftrightarrow$$

$f'(x) = 0$

f STEIGT MONOTON, ABER NICHT STR. MONOTON

f FÄLLT MONOTON, ABER NICHT STR. MONOTON

$$\boxed{1} \cdot x^2 + p \cdot x + q = 0$$

$$\Rightarrow x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

$$f'(x) = \boxed{\frac{1}{4}} \cdot x^2 - 2x + 3 \overset{!}{=} 0 \Leftrightarrow x^2 - 8 \cdot x + 12 = 0$$

$p = \;, q = 0$

$$\Rightarrow \quad x_{1,2} = -\frac{(-8)}{2} \pm \sqrt{\left(\frac{-8}{2}\right)^2 - 12} = 4 \pm \sqrt{16 - 12}$$

$$= 4 \pm 2$$

$$\boxed{x_1 = 2 \;,\; x_2 = 6}$$

$$f''(x) = \frac{1}{2} \cdot x - 2$$

$$f''(2) = \frac{1}{2} \cdot 2 - 2 = -1 < 0$$

$$f''(6) = \frac{1}{2} \cdot 6 - 2 = +1 > 0$$

BEI 2 LIEGT REL. MAX. VOR
-"- 6 -"- MIN. VOR.

$$f(2) = \frac{1}{12} \cdot 2^3 - 2^2 + 3 \cdot 2 = \frac{1}{12} \cdot 8 - 4 + 6 = \frac{2}{3} + 2 = \underline{\underline{2\tfrac{2}{3}}}$$

$$f(6) = \frac{1}{12} \cdot 6^3 - 6^2 + 3 \cdot 6 = \frac{{}^{3}\cancel{6} \cdot 6 \cdot \cancel{6}}{\cancel{2} \cdot \cancel{6}} - 36 + 18 = 18 - 18 = 0$$

$\boxed{(2;\ 2\tfrac{2}{3}) \text{ REL. MAX} \quad (6;0) \text{ REL. MIN.}}$

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$$f(x) = \frac{1}{12} \cdot x^3 - x^2 + 3x$$

$$\lim_{x \to +\infty} f(x) = +\infty \; ; \; \lim_{x \to -\infty} f(x) = -\infty$$

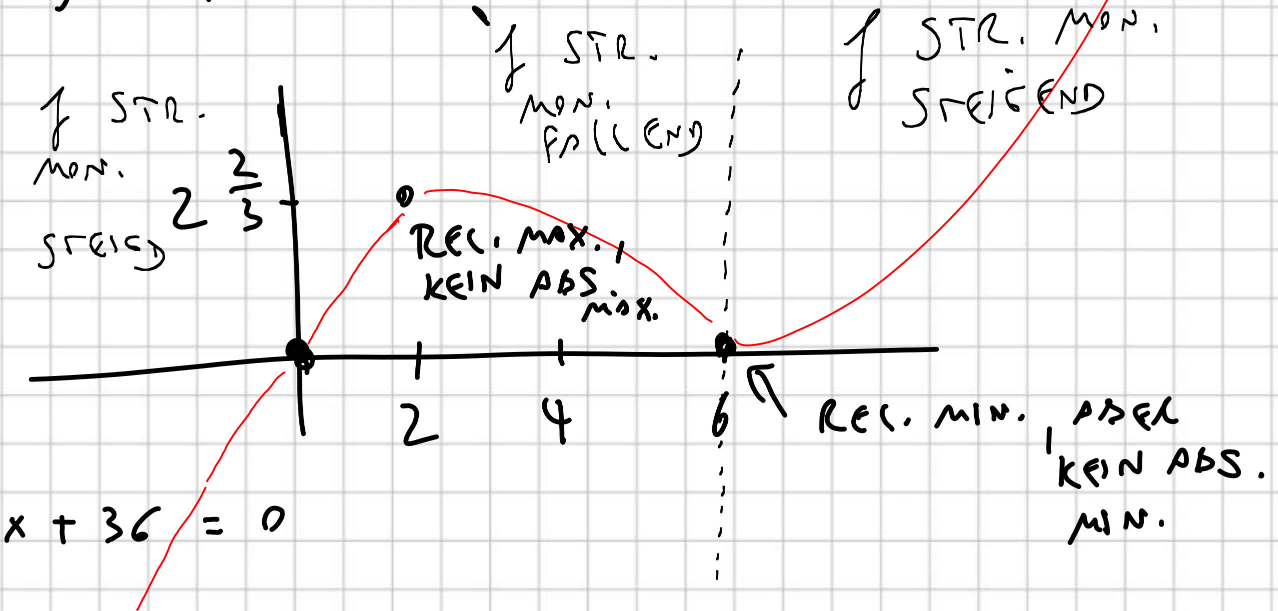
NST

$x = 0$

$f(x) = 0$

$\Leftrightarrow x \cdot \left(\frac{1}{12} \cdot x^2 - x + 3\right) = 0$

$\frac{1}{12} \cdot x^2 - x + 3 = 0 \Leftrightarrow x^2 - 12x + 36 = 0$



$$\Leftrightarrow \quad x_{1,2} = -\frac{(-12)}{4} \pm \sqrt{\left(\frac{-12}{2}\right)^2 - 36} = 6 \pm \sqrt{36-36} = 6$$

KRÜMMUNGSVERH.

$f''(x) \geq 0 \; \ldots \; f$ KONVEX

$f''(x) \leq 0 \; \ldots \; f$ KONKAV

$$f''(x) = \frac{1}{2}\cdot x - 2 \geq 0 \Leftrightarrow \frac{1}{2}\cdot x \geq 2 \Leftrightarrow \boxed{x \geq 4} \quad f \text{ KONVEX}$$

$x < 4 \; \ldots \; f$ KONKAV

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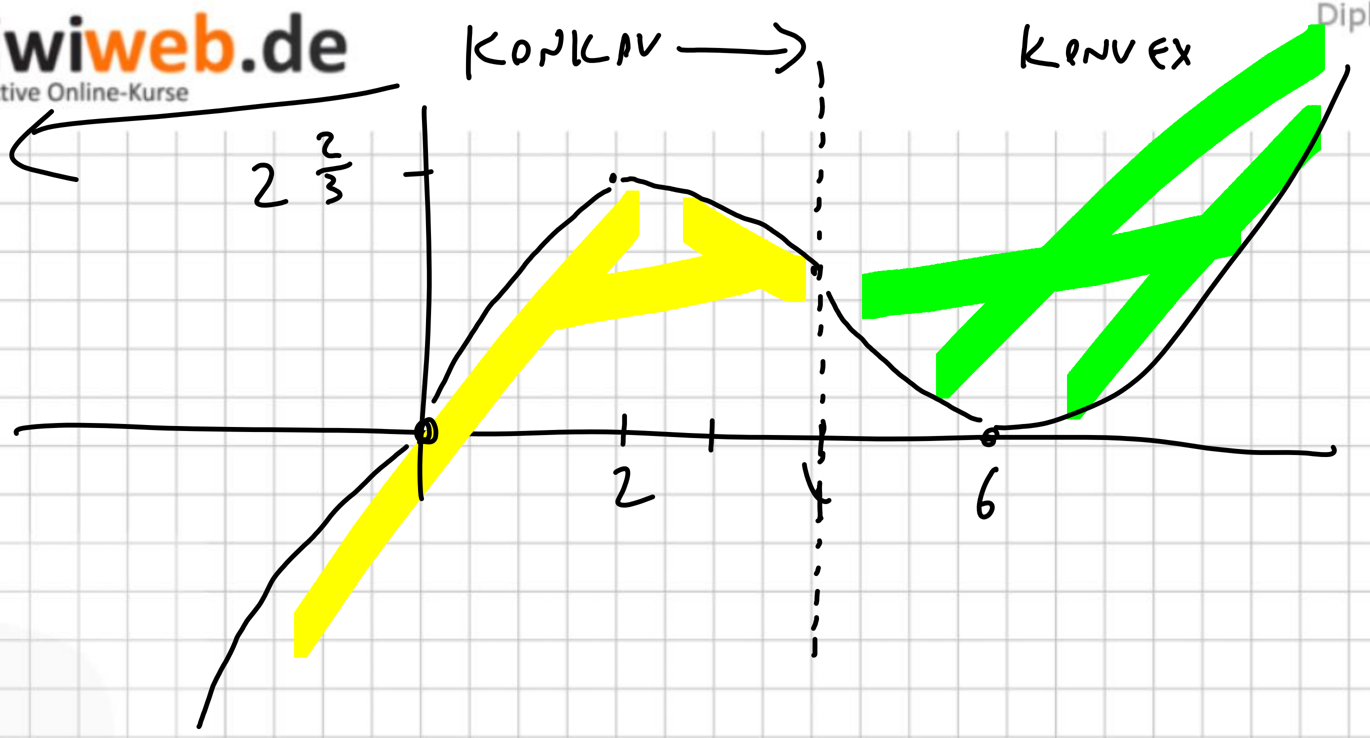

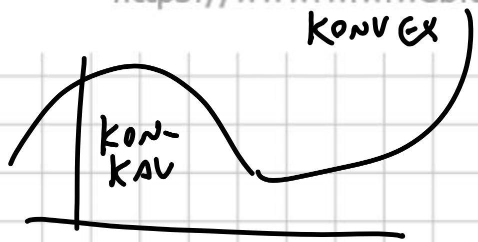

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$$f(x) = x \cdot e^{-x+1} \quad , \; x \geq 0$$

$$D_f(x) = \mathbb{R}_+$$

$$W_f(x) = \mathbb{R}_+$$

$$\boxed{(v \cdot w)' = v' \cdot w + v \cdot w'}$$

$$f'(x) = (\underbrace{x}_{v} \cdot \underbrace{e^{-x+1}}_{w})' = \underbrace{1}_{v'} \cdot \underbrace{e^{-x+1}}_{w} + \underbrace{x}_{v} \cdot \underbrace{\underbrace{e^{-x+1}}_{\text{ä.A.}} \cdot \underbrace{(-1)}_{\text{i.A.}}}_{w'}$$

$$= e^{-x+1} - x \cdot e^{-x+1} = \underbrace{e^{-x+1}}_{>0} \cdot (1-x)$$

$$f'(x) = 0 \Leftrightarrow 1 - x = 0 \Leftrightarrow \boxed{x = 1}$$

HIER KÖNNTE REL. EXTR. VORLIEGEN

$$f''(x) = e^{-x+1} \cdot (-1) \cdot (1-x) + e^{-x+1} \cdot (-1)$$

$$= e^{-x+1} \cdot (-1 + x - 1) = (-2 + x) \cdot e^{-x+1}$$

$$f''(1) = (-2+1) \cdot e^{-1+1} = -1 \cdot e^{0} = \underline{\underline{-1}} < 0$$

1 IST REL. MAX. STELLE

$$f(1) = 1 \cdot e^{-1+1} = 1 \cdot 1 = \underline{\underline{1}}$$

GRENZWERTE

$$\lim_{x \to 0} \left(x \cdot e^{-x+1} \right) = 0$$

$$\lim_{x \to \infty} \left(x \cdot e^{-x+1} \right) = 0$$

$$x \cdot e^{-x+1} = \frac{x}{e^{x-1}} \xrightarrow{x \to \infty} \frac{\infty}{\infty}$$

DE L'HOSPITAL

$$\frac{1}{e^{x-1}} \xrightarrow[\infty]{x} \frac{1}{\infty} = 0$$

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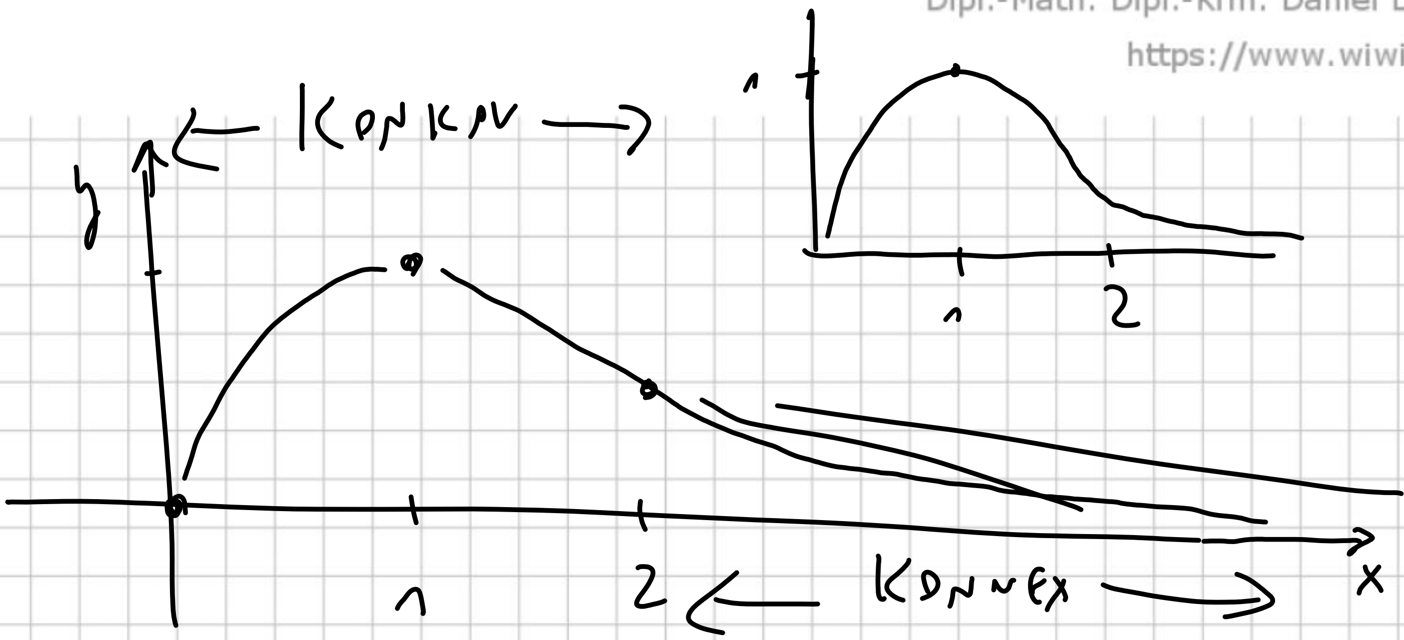

$f''(x) \geq 0$

$\Leftrightarrow (x-2) \cdot \underbrace{e^{-x+1}}_{>0} \geq 0 \Leftrightarrow x-2 \geq 0 \Leftrightarrow \boxed{x \geq 2}$

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